Newton’s Laws in a Messy World:
- Friction.
- Inclined plane.
- How to pick the right basketball shoes: PHYS 2A approach.
- Drag force. Terminal speed.
- The physics of skydiving.
- Pulleys.
- Equilibrium.
- Camping with bears.

Static friction

- Direction of the static friction force?
  - opposite to the direction of the applied force that is attempting the motion.

\[ f_s \leq \mu_s F_N \]

Object at rest.

Molecular origin of friction

Applied force

Friction force

Directions:
- \( F_N \) normal force
- \( F_g \) gravitational force
- \( f_s \) static friction
- \( f_f \) kinetic friction
- \( F_a \) applied force
Kinetic friction

- Before you can get an object to move you must overcome the maximum static friction.

- Once you have an object moving over a surface, the friction will become kinetic friction, $f_k$.

- $f_k < f_{s, \text{max}}$ for a given surface.

- Magnitude of kinetic friction:

\[
f_k = \mu_k F_N
\]

(Unlike static friction, $f_k$ is independent of the value of any applied force!) 

Object started moving

Static and kinetic friction: summary

<table>
<thead>
<tr>
<th>Friction force</th>
<th>Applied force</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_s, \text{max} = \mu_s F_N$</td>
<td>$F$</td>
</tr>
<tr>
<td>$f_s = F$</td>
<td>$F$</td>
</tr>
<tr>
<td>$f_k = \mu_k F_N$</td>
<td>$F$</td>
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$\Rightarrow$ when calculating any type of friction, it is helpful to calculate the normal force.

Object at rest
Object started moving
**Coefficient of kinetic friction**

Typical $\mu_k$ range from 0.01 (smooth/well-lubricated surf.) to 1.5 (very rough surfaces)

- Rubber on dry concrete: $\mu_k = 0.8$ $(\mu_s = 1)$
- Dry bone on bone: $\mu_k = 0.3$
- With synovial fluid: $\mu_k = 0.003$

**Example**

An applied 12N horizontal force pushes a block weighing 5.0N against a vertical wall. The coefficient of static friction between the wall and the block is 0.60, and the coefficient of kinetic friction is 0.40. Assume that the block is not moving initially. Will the block ever move?

**Solution**

- First, define a coordinate system.
- Let’s choose up as $+y$ and the direction of the applied force as $+x$. 
Normal Force

Solution (cont’d)

• Next, draw a force diagram for the block:

\[ F_{\text{friction, wall on block}} \]
\[ F_{\text{normal, wall on block}} \]
\[ F_{\text{gravity, Earth on block}} \]

\[ F_{\text{applied, you on block}} \]

• No need to break the forces into components => turn to Newton’s Laws… Which one?
• Know the block doesn’t accelerate in the x-dir., but not sure about the y-dir.

Normal Force

Answer

• Apply Newton’s 1st Law in the x-direction:

\[ \Sigma F_x = 0 \]
\[ F_{\text{applied}} - F_{\text{normal}} = 0 \]
\[ F_{\text{normal}} = F_{\text{applied}} = 12 \text{N} \]

• If the block is not going to move in y-dir. => \( \Sigma F_y = 0 \)

\[ f_{s, \text{max}} \geq f_s = F_{\text{grav}} = \text{weight} \]

=> condition for the block not to move:

\[ \mu_s F_{\text{normal}} = f_{s, \text{max}} \geq F_{\text{grav}} = 5.0 \text{ N} \]

\[ f_{s, \text{max}} = 0.60 \cdot 12 \text{N} = 7.2 \text{ N} > F_{\text{grav}} \]

=> the block will not move.
Attached Masses

- If masses are attached by ropes/strings => tensions is involved.

![Attached Masses Diagram]

- By Newton’s Third Law: $T_{m_2 \text{ on } m_1} = T_{m_1 \text{ on } m_2}$ (but opposite in direction).

- Assuming the rope cannot be stretched (inextensible): $a_{1x} = a_{2x} = a_x$

Inclined Plane

**Example**

Rachel, a PHYS 2A student, is at the shoe store to buy a pair of basketball shoes that have the greatest traction on a specific type of hardwood. To determine the coefficient of static friction, $\mu_s$, Rachel places each shoe on a plank of wood and tilts the plank to an angle $\theta$, at which the shoe just starts to slide. Obtain an expression for $\mu_s$ as a function of $\theta$.

**Solution**

- First, define a coordinate system.
**Inclined Plane**

**Solution (cont')**

- Make a clever choice of coordinate system:
  angled ⇒ only need to break one force into components.

- Force diagram for the shoe:

- Break \( F_{\text{grav}} \) into x and y components:
  \[ F_{gx} = mg \sin \theta \quad F_{gy} = mg \cos \theta \]
  Note similar triangles: angles are congruent and sides are in proportion.

- Apply the appropriate Newton’s Laws. Shoe at rest ⇒ apply First Law, separately in x and y directions:
  \[ \Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0 \]

**Answer**

- For the x-direction:
  \[ \Sigma F_x = F_{gx} - f_s = 0 \]
  \[ f_s = F_{gx} = mg \sin \theta \]

- For the y-direction:
  \[ \Sigma F_y = F_N - F_{gy} = 0 \quad \Rightarrow \quad F_N = F_{gy} = mg \cos \theta \]

- The shoe is on the verge of sliding ⇒ \( f_s = f_{s, \text{max}} \):
  \[
  f_{s, \text{max}} = \mu_s F_N \quad \Rightarrow \quad \mu_s = \frac{f_{s, \text{max}}}{F_N}
  \]
  \[ \mu_s = \frac{mg \sin \theta}{mg \cos \theta} = \tan \theta \quad \text{← unitless} \]
Drag Force

• The drag force, $F_D$, opposes motion in any fluid.

• For a limited but important range of objects and speeds,

$$F_D = \frac{1}{2} C \rho A v^2$$

- Drag coefficient.
- Can be lower for rough surfaces!
- Density of the fluid ($\rho_{air} = 1.2 \text{kg/m}^3$)
- Cross-sectional area (⊥ to velocity). E.g., for a sphere $A = \pi r^2$ (NOT the surface area!).

• The direction of the drag force is opposite to the direction of the velocity of the object.

Terminal speed

• Drop an object from the top of a building => it accelerates <= gains speed => drag force increases.

• Eventually, $v$ will increase to the point where $F_D = F_g$ and there will be no more acceleration.

• This speed is called terminal speed, $v_t$.

$$F_D = F_g, \quad \frac{1}{2} C \rho A v_t^2 = mg$$

$$v_t = \sqrt{\frac{2mg}{C \rho A}}$$

when $v = v_t$
Physics of Skydiving

\[ v_t = \sqrt{\frac{2mg}{C \rho A}} \]

Changing position of arms/legs skydivers can change \( C \) and \( A \) => adjust \( v_t \) of their fall.

Dropped separately, these divers have to change their \( v_t \) in order to arrive at same location at same time and link up!

Pulleys

Pulleys change the direction of the tension force in ropes.

\( \Rightarrow \) the tension for two masses may be in the same direction...

..or a tension force on one mass in the vertical direction may have a Third Law Pair in the horizontal direction.
Pulleys

- biomechanics; regulating weight-driven clock mechanism, friction...
- 200 years before “force” was even defined by Newton

Leonardo Da Vinci (1452-1519)

Pulley (Atwood’s Machine) (1784)

Example

- Two objects with masses 2.00kg \( (m_1) \) and 6.00kg \( (m_2) \) are connected by a light string that passes over a frictionless pulley. Determine the acceleration of each mass and the tension in the string.

Solution

- First, define a coordinate system.
- Let’s choose up as positive \( y \).
Atwood’s Machine

Solution (cont’d)

• Draw a force diagram for each mass separately.

• Forces are already broken up into components. Apply Newton’s 2nd Law separately to each object.

Atwood’s Machine

Solution (cont’d)

• Unstretchable string ⇒ \( a_1 = -a_2 \), \( a_1 = a_2 = a \)

• Tensions have the same magnitude, \( T \)

• For the 2kg mass (\( m_1 \)):

\[
\Sigma F_y = m_1a_1y \quad T - m_1g = m_1a
\]

\( T = m_1a + m_1g \)

• For the 6kg mass (\( m_2 \)):

\[
\Sigma F_y = m_2a_2y \quad T - m_2g = m_2(-a)
\]

\( m_1a + m_1g = m_2g - m_2a \)

\( T = m_2g - m_2a \)

\[ a = g(m_2 - m_1)/(m_2 + m_1) \]
**Atwood’s Machine**

**Answer**

\[ a = g(m_2 - m_1)/(m_2 + m_1) \]

- Inserting the values:
  \[ a = (9.80\text{m/s}^2)(6\text{kg} - 2\text{kg})/(6\text{kg} + 2\text{kg}) \]
  \[ = 4.90\text{m/s}^2 \]

- Recall our equation for tension from Newton’s 2nd Law (either one):
  \[ T = m_2g - m_2a \quad T = m_2(g - a) \]
  \[ T = 6\text{kg}(9.80\text{m/s}^2 - 4.90\text{m/s}^2) = 6\text{kg}(4.90\text{m/s}^2) = 29.4\text{N} \]
  - the tension in the entire string.

**For Next Time:**

- Study HARD for Quiz 3 (Ch.4)
- Read Chapter 5
- Do HW for Chapter 5
- Study for Quiz 4: Ch.5
- Attend Problem & Discussion sessions: we hold them exclusively FOR YOU!