Today:
Motion in more than one dimension.
Projectile motion.
Circular motion.

Independence of vertical and horizontal motions

- The kinematics equations that we derived for 1D motion (with $a = \text{const}$) hold in 2D.
- But the equations apply separately to each component of 2D motion.
- Big physics concept:

\[ \text{Perpendicular components of motion are independent of each other.} \]

$\Rightarrow$ Giving an object velocity or acceleration in the horizontal direction does not affect how fast it falls to the ground.
Kinematics Equations in 2D

• for motion in 2D the individual components of motion obey the following equations as long as \( a_x \) and \( a_y \) are constants:

\[
\begin{align*}
v_x &= v_{0x} + a_x t \\
\Delta x &= \frac{1}{2} (v_{0x} + v_x) t \\
\Delta x &= v_{0x} t + \frac{1}{2} a_x t^2 \\
v_{x}^2 &= v_{0x}^2 + 2a_x \Delta x
\end{align*}
\]

\[
\begin{align*}
v_y &= v_{0y} + a_y t \\
\Delta y &= \frac{1}{2} (v_{0y} + v_y) t \\
\Delta y &= v_{0y} t + \frac{1}{2} a_y t^2 \\
v_{y}^2 &= v_{0y}^2 + 2a_y \Delta y
\end{align*}
\]

• Note: subscripts now denote variables corresponding to the \( x \) and \( y \) directions (except for \( t \) - the link btw the two sets of equations).

Projectile Motion

• Projectile is an object that is launched into the air and then moves predominantly under the influence of gravity.

• We will neglect air resistance.

• A projectile follows a parabolic path (will prove this soon)
Typical projectile motion problem

A cannonball is shot at a given angle \((30.0^\circ)\) with a certain muzzle velocity \((100.0 \text{ m/s})\). How far from the cannon does the cannonball land if it lands at the same height that it is launched?

Solution

- First, define a coordinate system.
- Choose the upward direction as \(+y\), and the horizontal direction the cannonball travels as \(+x\).

Solution (cont’d)

- To solve, separate into two problems \((x\) and \(y)\).
- Break velocity into components.

\[
v_{0y} = |v_0| \sin \theta = (100 \text{ m/s}) \sin 30^\circ = (100 \text{ m/s}) \frac{1}{2} = 50.0 \text{ m/s}
\]

\[
v_{0x} = |v_0| \cos \theta = (100 \text{ m/s}) \cos 30^\circ = (100 \text{ m/s}) \frac{\sqrt{3}}{2} = 86.6 \text{ m/s}
\]

- Use the kinematics equations separately in each direction.
Projectile Motion

Solution (cont’d)

• Let’s try the x-direction. List the quantities we know:
  \( v_{ox} = +86.6 \text{ m/s} \)
  \( a_x = 0 \text{ m/s}^2 \)  = constant

• Don’t know: \( v_x, t, \Delta x \) (finding)

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• 3rd eq.: \( \Delta x = v_{ox}t + \frac{1}{2}a_x t^2 \)  =>  \( \Delta x = v_{ox}t \)

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• Other eqs.?

  \[ v_x = v_{ox} + a_x t \]  =>  \[ v_x = v_{ox} \]

  \[ \Delta x = \frac{1}{2}(v_{ox} + v_x)t \]  =>  \[ \Delta x = v_{ox}t \]

  \[ v_x^2 = v_{ox}^2 + 2a_x \Delta x \]  =>  \[ v_x^2 = v_{ox}^2 \]

⇒ turn to the y-direction.

Solution (cont’d)

• List the quantities we know for y-dir.:
  \( v_{oy} = +50.0 \text{ m/s} \)
  \( a_y = 9.80 \text{ m/s}^2 \)  = constant
  \( \Delta y = 0 \)  (only if falls to the same height)

• Don’t know: \( v_y, t \)

• What is the variable we can solve for in the y-dir. and substitute in the x-dir. equations: ? time, t

• 3rd eq.: \( \Delta y = v_{oy}t + \frac{1}{2}a_y t^2 \)  =>  \( t_{1,2} = \frac{-v_{oy} \pm \sqrt{v_{oy}^2 + 2a_y \Delta y}}{a_y} \)

  \( t_1 = 0 \)  the time when the ball was first shot

  \( t_2 = \frac{2v_{oy}}{a_y} \)  the time is takes to hit the ground

  \( t_2 = \frac{2 \times 50.0 \text{ m}}{9.80 \text{ m/s}^2} = 10.2 \text{ sec} \)
**Projectile Motion**

**Answer**

- Substitute $t$ into the x-dir. equation:
  
  $\Delta x = v_{0x} \cdot t$

  $\Delta x = (86.6 \text{m/s})(10.2 \text{ sec}) = 883 \text{ m}$

- Other typical questions:
  How high does it go?
  How long is it in the air?
  Projectiles with $\Delta y \neq 0$

**Clicker Question**

A cannonball is shot from a cannon with a muzzle velocity of 100 m/s at an angle of 30° with respect to the horizontal. Which of the following statements is correct concerning the resulting projectile motion?

A) The speed is zero at the top of its path.
B) The speed is a minimum at the top of its path.
C) The speed is a maximum at the top of its path.
D) The speed is 9.8 m/s at the top of its path.
E) The acceleration is zero at the top of its path.
Projectile Motion

- At any point in the path of a projectile, the speed (i.e., the magnitude of the projectile’s velocity) is
  \[ v = \sqrt{v_x^2 + v_y^2} \]
  \( \Rightarrow \) minimum at the top

...and the angle that the velocity makes with the horizontal is

\[ \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) \]

Shape of a projectile’s trajectory

- Describe the vertical position \( y \) of a projectile as a function of horizontal position \( x \).
- If it indeed follows a parabolic path, then \( y \) will be a quadratic function of \( x \).
- Consider a projectile launched from origin \( (x_0=y_0=0) \) with initial speed \( v_0 \) at an initial angle \( \theta_0 \) to the horizontal.
- Choose +x and +y directions
**Shape of a projectile’s trajectory**

- 3rd kinematics eqs.: \( \Delta x = v_0x t + \frac{1}{2} a_x t^2 \) \( \Delta y = v_0y t + \frac{1}{2} a_y t^2 \)
- => the horizontal position, \( x \), at any given time is
  \[ x-x_0 = v_0x t \Rightarrow x = v_0x t = v_0 \cos(\theta_0) t \]
- The vertical position, \( y \), at any given time is
  \[ y = v_0y t + \frac{1}{2} a_y t^2 = v_0 \sin(\theta_0) t - \frac{1}{2} g t^2 \]
- As almost always with projectile motion, solve the x-eq. for \( t \) to input into the y-eq.:
  \[ t = \frac{x}{v_0 \cos(\theta_0)} \Rightarrow y = v_0 \sin(\theta_0) \left( \frac{x}{v_0 \cos(\theta_0)} \right) - \frac{1}{2} g \left( \frac{x}{v_0 \cos(\theta_0)} \right)^2 \]
- \( y \propto x^2 \)
- Neglected air resistance

**The range of a projectile**

- How far will it move horizontally over level ground?
- Start with the trajectory equation, find out when \( y = 0 \) (assumes level ground !!):
  \[ y = x \tan(\theta_0) - \frac{g x^2}{2v_0^2 \cos^2(\theta_0)} \]
- Answer #1: \( x = 0 \) - at the beginning of the motion.
- Answer #2: occurs when \((\ldots)=0 \Rightarrow \tan(\theta_0) = \frac{g x}{2v_0^2 \cos^2(\theta_0)} \)
  \[ x = \frac{2v_0^2 \cos^2(\theta_0) \tan(\theta_0)}{g} = \frac{2v_0^2 \cos(\theta_0) \sin(\theta_0)}{g} \]
- \( \sin 2\theta = 2 \sin \theta \cos \theta \)
- only valid for level ground

\[ x = \frac{v_0^2}{g} \sin(2\theta_0) \]
Horizontal range

\[ x = \frac{v_0^2}{g} \sin(2\theta_o) \]

- Min range? \( \theta = 0, 90^\circ \).
- Max range for a given \( v_0 \)? \( \theta = 45^\circ \).
- \( \theta < 45^\circ \): \( v_x \) is greater but \( y_{\text{max}} \) is smaller \( \Rightarrow \) isn't in the air as long
- \( \theta > 45^\circ \): \( y_{\text{max}} \) is greater \( \Rightarrow \) longer in the air, but \( v_x \) is smaller

Example

A pirate ship is 560 m from a fort defending the harbor entrance of an island. A defense cannon at the fort, located at sea level, fires cannonballs at initial speed \( v_0 = 82 \text{ m/s} \). At what angle \( \theta_o \) from the horizontal must a cannonball be fired to hit the ship?

Solution

- We derived the Range Equation assuming that up is \( +y \) and the direction of cannonball’s motion is \( +x \).

\[ x = \frac{v_0^2}{g} \sin(2\theta_o) \quad \Rightarrow \quad 2\theta_o = \sin^{-1}\left(\frac{xg}{\frac{v_0^2}{2}}\right) \quad \Rightarrow \quad \theta_o = \frac{1}{2} \sin^{-1}\left(\frac{xg}{\frac{v_0^2}{2}}\right) \]

\[ \theta_o = \frac{1}{2} \sin^{-1}\left(\frac{(560 \text{ m})(9.8 \text{ m/s}^2)}{(82 \text{ m/s})^2}\right) = \frac{1}{2} \sin^{-1}(0.816) = \frac{1}{2} (54.7^\circ) = 27^\circ \]
Answer

• But we are not done, as there are two possible values for the angle $\theta_o$ that satisfy $\sin(2\theta_o) = 0.816$.

54.7° and $(180° - 54.7°) = 125.3°$

$\theta_o = \frac{1}{2}(125.3°) = 63°$

A ball must be fired at an angle 27° or 63° from the horizontal to hit the ship.

Warning: When using the range equation, watch out for the $2\theta$ part (could give trouble).

For Next Time (1)

• Read Chapter 3
• Do homework for Chapter 3
• Study for Quiz 2 (Ch. 3)

Quiz 1 (Ch.1,2): Friday 4/12, 5:00-5:50 PM, Galbraith 242

• Know your 3-digit quiz code by heart.
• Arrive at least 5 min before start (i.e., by 4:55 pm).
• Closed book exam. You may bring a half-a-standard-page of notes (OK to write on both sides).
• Bring a scantron No. F-289-PAR-L (red) & #2 pencil.
• Bring standard calculator. No laptop, no cell phone etc.
• Bring your picture ID – proctors will check identity.
• Academic integrity rules will be rigorously enforced.
For Next Time (2)

• iclicker registration deadline: Monday, April 15

Register on www.iclicker.com

These clickers have **NOT** yet been REGISTERED:

- #A9001188
- #A937852B
- #AA1157EC
- #AA4801ED
- #AA066617
- #AA4F44B7
- #AA4F969A
- #AABFE85E1
- #AAB3351C9
- #AB42B663
- #AC2263ED
- #AC3153CE
- #AEC0A444
- #AFOF288B
- #AF108639
- #AF113D965
- #AF2663EA
- #AF2E20DA1
- #AF2E56D7
- #AF32C558
- #AF389106
- #AF3D3240
- #AF53E21C
- #AF5256A8
- #AF64418A
- #AF6F76B6
- #AF7312CE
- #AF7F5F21
- #AF80E8C7
- #AF821D30
- #BF0C0B94