Lecture 11

Today:

Conservation of mechanical energy.

Test of conservation of mechanical energy!

Force and potential energy.

Potential energy curves.

Potential Energy

- Kinetic energy $K$ quantifies motion.
- Potential energy $U$ is associated with the relative position of objects that exert forces on each other.
- Potential energy is the “stored work” that can be recovered as kinetic energy.
- SI units of $U$: joules.
- $U$ is also a state variable: $\Delta U$ only depends on the initial and final values (it is not path-dependent).
**Potential Energy and Work**

- Potential energy defined in terms of the work:

  \[
  \Delta U = -W
  \]

  \(\Delta U\) associated with the force \(W\) done by the force

- For gravitational force near Earth’s surface:

  \[
  \Delta U = -\int \vec{F}_g \cdot d\vec{r} = -\int (-mg) \hat{j} \cdot dy
  \]

  \(\Delta U = mg(y_f - y_i) = mg\Delta y\)

**Conservative and nonconservative forces**

- Conservative force:
  “gives back” work that has been done against it.

- Nonconservative force: the work done against it cannot be recovered as \(K\).

- How can we tell if a force is conservative or not? Have the force act over a **closed** path:

  If force is **conservative** (gravity on climber):

  \[
  W_{AB} + W_{BA} = 0
  \]

  \(= -mg\Delta y = mg\Delta y\)

  If nonconservative (friction force on chair):

  \[
  W_{AB} + W_{BA} \neq 0
  \]

  \(= -\mu N\Delta x = -\mu N\Delta x\)
Properties of Conservative Forces

• A conservative force is the one that does zero work on an object moved around any closed path:

\[ \int \vec{F} \cdot d\vec{r} = 0 \]

• Related property: work done by a conservative force is path-independent (depends only on the endpoints).

• Conservative forces: gravity, electrostatic forces, spring force (fundamentally electric).

• Nonconservative forces: friction, air drag.

Conservation of Mechanical Energy

• Define mechanical energy as \( E_{\text{mec}} = K + U \)
• Recall work-energy theorem: \( \Delta K = W_{\text{net}} \)

\[ \Delta K = W_c + W_{nc} = - \Delta U + W_{nc} \quad \Rightarrow \quad \Delta K + \Delta U = W_{nc} \]

\[ \Delta U = - W_c \]

\[ \Rightarrow \text{if} \ W_{nc} = 0 \quad \text{then} \quad \Delta E_{\text{mec}} = 0 \quad \Leftrightarrow \quad E_{\text{mec}} = \text{const} \]

If a system is subject only to conservative forces, its mechanical energy \( E_{\text{mec}} = K + U \) remains constant.

\[ \Rightarrow \text{In the absence of nonconservative forces,} \]

\[ K_1 + U_1 = K_2 + U_2 = \text{const} \]

for two time moments 1 and 2.
Conservation of mechanical energy demo

How far will it go?

Example
A block with an initial velocity of 3.0 m/s slides across a horizontal frictionless floor and then up a frictionless 30° ramp. What distance will it travel up the ramp?

Solution
• First, define a coordinate system.
• Choose upward as +y with y = 0 on the floor.
Solution

- No nonconservative forces $\Rightarrow$ $E_{\text{mech}}$ conserved.

- While on the floor:
  \[ E_{\text{floor}} = U_1 + K_1 = (1/2)mv_0^2 \]

- When at its max height:
  \[ E_{\text{top}} = U_2 + K_2 = mg\Delta y \]
  \[ E_{\text{floor}} = E_{\text{top}} \Rightarrow \frac{1}{2}mv_0^2 = mg\Delta y \Rightarrow \Delta y = \frac{v_0^2}{2g} \]

\[
\begin{align*}
d &= \frac{\Delta y}{\sin 30^\circ} = \frac{v_0^2}{2g \sin 30^\circ} = \frac{(3.0 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)0.5} = 0.92 \text{ m}
\end{align*}
\]

Clicker Question

Three rocks of equal mass are thrown with identical speeds from the same height. Rock A is thrown vertically downward, rock B is thrown vertically upward, rock C is thrown horizontally. Assume air resistance is negligible. Which rock has the greatest speed just before it hits the ground?

A) Rocks A, B and C hit the ground with the same speed.
B) Rock A.
C) Rock B.
D) Rock C.
E) Both Rock A and Rock C.
Spring Potential Energy

• For every conservative force that can perform work there is a potential energy that can be placed in the conservation of mechanical energy equation.

• Nonconservative forces do not have potential energies associated with them.

• If an ideal spring is stretched (or compressed):

\[
\Delta U = -W = -\int_{x_i}^{x_f} F_{\text{spring}} \cdot dr = -\int_{x_i}^{x_f} (-kx) \cdot dx = \int_{x_i}^{x_f} (kx)dx = k \frac{x^2}{2}\bigg|_{x_i}^{x_f}
\]

\[
\Delta U = k \frac{x_f^2 - x_i^2}{2} \quad \text{--- Spring Potential Energy}
\]

Nonconservative forces and Mechanical Energy

• In a system where nonconservative forces can cause energy changes, \(E_{\text{mec}}\) is not conserved.

• Generalized principle of conservation of mech. energy:

\[
\Delta E_{\text{mec}} = \Delta K + \Delta U = W_{\text{nc}}
\]

where \(W_{\text{nc}}\) is the work done by nonconservative forces.

• When starting energy problems, the first question to ask is: Are nonconservative forces doing work on the system?
Example

Mechanical Energy

A 10.0 kg block is released from point A. The track is frictionless except for the portion between points B and C of a length 6.00 m. The block travels down the track, hits a spring (spring constant 2,250 N/m), and compresses the spring 0.300 m from its equilibrium position before coming to rest momentarily.

Determine the coefficient of kinetic friction, \( \mu_k \), between the block and the rough surface.

Solution

Mechanical Energy

- Choose up as +y with the lowest point y = 0.
- Nonconservative forces?
  - Friction btw B and C.
    \[
    \Delta E_{\text{mec}} = \Delta K + \Delta U = W_{\text{nc}} = W_{\text{friction}}
    \]
    Block starts at rest and ends at rest.
    \[
    \Delta U = \Delta U_{\text{grav}} + \Delta U_{\text{spring}} = W_{\text{friction}}
    \]
    \[
    mg\Delta y + \frac{1}{2}kx^2 = F_{\text{frict}}\cdot d_{BC}
    \]

- Block does not accelerate in y-dir. btw B and C =>
  \[
  \Sigma F_y = F_N - F_g = 0 \quad \Rightarrow \quad F_N = F_g = mg
  \]
  \[
  F_{\text{frict}} = \mu_k F_N \quad \Rightarrow \quad \mu_k = \frac{F_{\text{frict}}}{F_N} = -\frac{mg \Delta y + kx^2 / 2}{d_{BC} mg}
  \]
  \[
  \mu_k = -\frac{(10\text{kg})(9.8\text{N/kg})(0m-3m) + (1/2)(2,250\text{N/m})(0.3m)^2}{(6m)(10\text{kg})(9.80\text{m/s}^2)} = 0.328
  \]
For Next Time:

- Read Ch. 6&7
- Finish up HW for Chapters 6 and 7
- Study VERY HARD for Quiz 5: Ch. 6, 7